7. THERMAL RADIATION

7.1. Kirchhoff's Assumption about Thermal Radiation

The electromagnetic radiation of a body in a state of thermodynamic equilibrium with the environment is called *thermal radiation*. The electromagnetic radiation of a body in a state of thermodynamic equilibrium with the environment is called thermal radiation. What is thermodynamic equilibrium? It looks like a train standing at the station: passengers move along the train, but the train does not move relative to the station. Thermodynamic equilibrium means that particles inside the body can move and exchange energy, but there are no flows. This activity on a microscale is averaged over the body, so that the energy and temperature of the body do not change.

Where does the thermal radiation of even a cold body come from? Let's recall that even in a piece of ice the size of a cherry there are billions of free electrons. They are accelerated and decelerated inside the substance, and such movement of charged particles leads to the generation of electromagnetic waves by them.

Thermal radiation is equilibrium radiation, because it is emitted by a body in a state of thermodynamic equilibrium. Nonequilibrium radiation appears if energy is transferred to the body from the outside. An example of nonequilibrium radiation is luminescence, when a body glow occurs when photons or electrons or other energy carriers act on it.

In 1859, Gustav Kirchhoff formulated a theorem in which it was stated that the thermal radiation of any body is determined by some universal function.

A year later, Kirchhoff introduced the concept of a perfect black body (or simply – a black body, BB) with an absorption coefficient $\alpha_{bb} = 1$ for all wavelengths. Fig. 7.1 shows the black body models. Models are close in properties to the black body, but they have $\alpha < 1$.

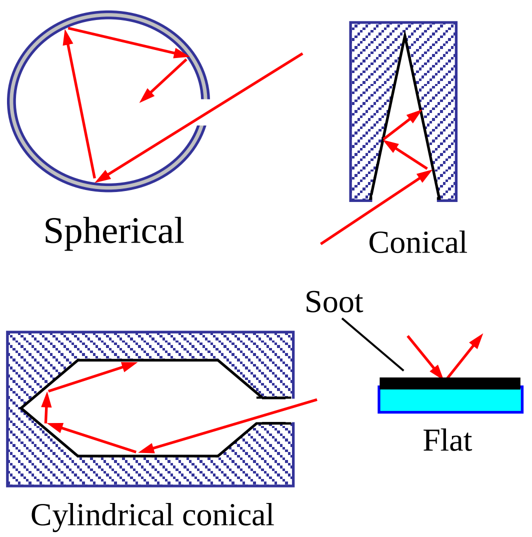

Fig.7.1. Models of a black body

Kirchhoff's theorem was later called the Kirchhoff radiation law. This law can be formulated as follows

$$\frac{p_{\lambda,T}}{\alpha_{\lambda,T}} = p_{bb}\left(\lambda, T\right),$$

where $p_{\lambda,T}$ is the spectral density of an equilibrium radiation of the body at a temperature T and at a wavelength λ, $\alpha_{\lambda,T}$ is the spectral absorption coefficient and $p_{bb}(\lambda, T)$ is the unknown universal function. Physicists had been looking for this function for 40 years.

7.2. Search for the Kirchhoff's Universal Function

Studying the properties of the Kirchhoff's universal function, the Austrian physicists Joseph Stefan (Fig. 7.2, *a*) and Ludwig Boltzmann (Fig. 7.2, *b*) found in 1884 the dependence of the total power of thermal radiation from the unit surface area of a BB at a temperature *T* (the Stefan-Boltzmann law):

$$p_{bb}\left(T\right) = \sigma T^4,$$

where σ is the Stefan-Boltzmann constant, $\sigma = 5.67 \cdot 10^{-8}\ \mathrm{W/m^2K^4}$. This dependence was derived by Stefan in 1879 on the basis of experimental data and in 1884 by Boltzmann on the basis of thermodynamic concepts of light pressure.

In 1893, the German physicist Wilhelm Wien showed that the universal function has a spectral maximum, which is determined only by body temperature (Wien's displacement law):

a *b*

Fig. 7.2. The Austrian physicists Joseph Stefan (1835–1893, *a*) and Ludwig Boltzmann (1844–1906, *b*)

$$\lambda_{\max}(T) = \frac{b}{T},$$

where b is the Wien constant, b = 2898 μm·K.

What practical benefits can be derived from these two laws? Let's look at two examples.

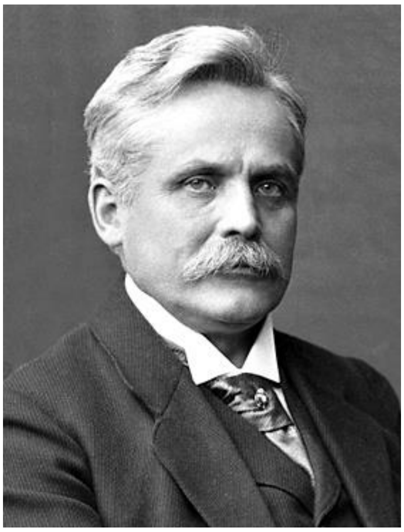

Fig. 7.3. The German physicist Wilhelm Wien (1864–1928)

Suppose we are interested in how many times M the power of thermal radiation of a kettle will increase when water is heated from a temperature of T_1 = 20°C to a boiling point of T_2 = 100°C. To apply the Stefan-Boltzmann formula to a real body, $p_{bb}(T)$ must be multiplied by the spectral absorption coefficient $\alpha_{\lambda,T}$ of the surface of this body and its area A. Answer:

$$M = \frac{\alpha_{\lambda,T} A\sigma T_2^4}{\alpha_{\lambda,T} A\sigma T_1^4} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{373}{293}\right)^4 = 2.6.$$

As you can see, in this case we are not interested in either the absorption coefficient of the kettle’s surface or its area.

Suppose you are faced with the task of measuring the temperature of molten metal, a very important parameter of the metallurgical process. How can you do that?

This can be done using a pyrometer, infrared thermometer that determines the color temperature T_c of a body from its emission spectrum, i.e. according to the Wien's formula $\lambda_{\max} = b/T_c$. The closer the properties of the measured body to the properties of the black body, the more similar their thermal radiation spectra become and the higher the accuracy of the measurement of body temperature.

The first device to measure temperatures in the range of 1000–3000 K was a disappearing-filament pyrometer invented in 1901. The operation of this device is based on a comparison of the color of the thermal radiation of the measured body and the heated filament, the image of which was observed against the background of the image of the body. By changing the current through the filament, achieve the disappearance of the filament image against the background of the body image, which indicates the equalization of temperature of the filament and the body. Calibration of the pyrometer is carried out using radiation from a blackbody model.

7.3. The Planck's Law of Thermal Radiation

In 1900, the German physicist Max Planck (Fig. 7.4) derived a formula for the spectral density of radiation from a unit surface area of a black body based on classical concepts (Planck's radiation law):

$$p_{bb}\left(\nu,T\right)=\frac{2\pi h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1},$$

where h is the Planck's constant, h = 6.63·10^{-34} J·s, ν is the frequency of radiation, and k is the Boltzmann's constant, k = 1.38·10^{-23} J/K. The same formula, expressed in terms of wavelength, has the form:

$$p_{bb}\left(\lambda,T\right)=\frac{2\pi hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1}.$$

This is the universal function that physicists have been looking for so long. A strange quantity h appears in the formula, which Planck called the quantum of action. This formula means that the thermal radiation of a black body emits in portions, quanta of energy. In a more general sense, this means that the exchange of energy between bodies occurs in portions that are multiples of the quantum of action h. A view of dependence of the relative spectral density of blackbody radiation (blackbody irradiance) on wavelengths at different temperatures is shown in Fig. 7.5.

Although Planck doubted the validity of his formula, experimental studies confirmed it. Planck's doubts are caused by the incompatibility of the obtained formula with classical electrodynamics (Maxwell equations) and, in general, with classical physics, in which energy is always a continuous quantity.

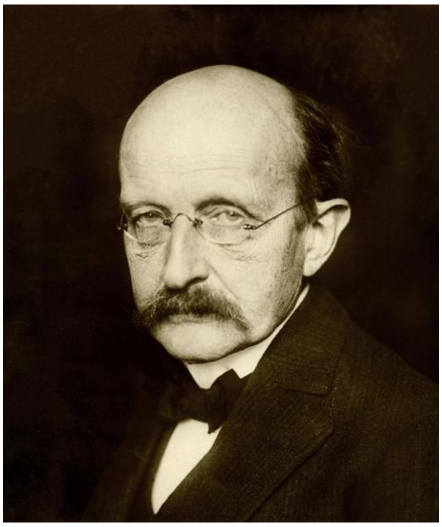

Fig. 7.4. The German physicist Max Planck (1858–1947)

The Stefan-Boltzmann law can be obtained by integration, and the Wien's law by differentiation the universal function with respect to λ:

$$p_{bb}\left(T\right)=\int_0^\infty p_{bb}\left(\lambda,T\right)d\lambda=\sigma T^4,$$

$$\lambda_{\max}(T)=\frac{\partial p_{bb}\left(\lambda,T\right)}{\partial\lambda}=\frac{b}{T}.$$

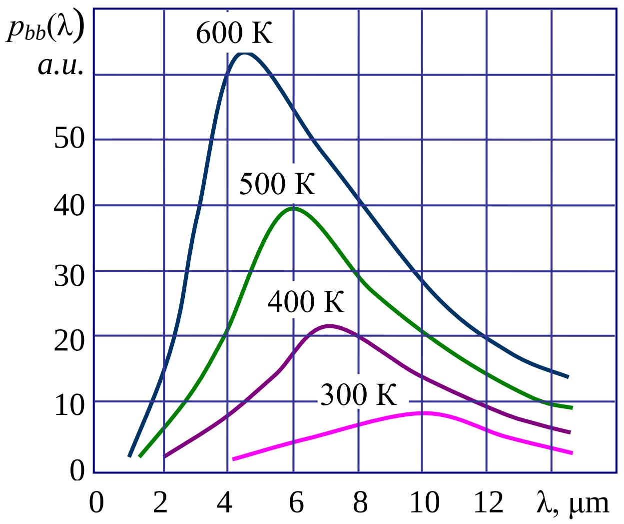

Fig. 7.5. Dependence of the relative blackbody irradiance on wavelengths at different temperatures (*a.u.* – arbitrary units)

Let's see what application the Planck's formula has in practice. Suppose we want to determine the power of thermal radiation of human body at $\lambda = 10$ μm within the spectral range $\Delta\lambda = 0.5$ μm.

The lower curve in Fig. 7.5 corresponds to the spectral characteristic of the thermal radiation of the human body. Assuming that the function $p_{bb}(\lambda)$ is practically unchanged in the spectral range , we can find the power of thermal radiation of the human body (and any other body) from the formula

$$P = \alpha_{\lambda,T}\, p_{bb}(\lambda) A\Delta\lambda ,$$

where $\alpha_{\lambda,T}$ is the spectral coefficient of absorption of the body's surface (let it be 0.3), and A is the area of the body (let it be 0.9 m^2). At a body temperature of $T = 310$ K, the power of thermal radiation in the indicated spectral range is

$$P = \frac{2\pi hc^2}{\lambda^5}\frac{\alpha_{\lambda,T} A\Delta\lambda}{\exp(hc/\lambda kT)-1} =$$

$$= \frac{2\cdot 3.14\cdot 6.63\cdot 10^{-34}\left(3\cdot 10^8\right)^2\cdot 0.3\cdot 0.9\cdot 0.5\cdot 10^{-6}}{\left(10^{-5}\right)^5\left[\exp\left(6.63\cdot 10^{-34}\cdot 3\cdot 10^8/10^{-5}\cdot 1.38\cdot 10^{-23}\cdot 310\right)-1\right]} = 4.9\ \text{W}.$$

Let us use this result to find how much of the thermal radiation power of the human body enters the lens of an infrared camera with a radius $r = 2$ cm located at a distance $L = 100$ m. From this distance, the human body can be regarded as a point source of radiation, uniformly radiating in all directions. Then a part of the total thermal radiation flux equal to the ratio of the area of the input aperture of the lens to the area of the sphere of radius L will fall on the lens. Therefore, the power of the radiation incident on the lens of infrared receiver

$$P_{rec} = \frac{\pi r^2}{4\pi L^2} P = \frac{0.02^2}{4\cdot 100^2} 4.9 = 4.9\cdot 10^{-8}\ \text{W}.$$

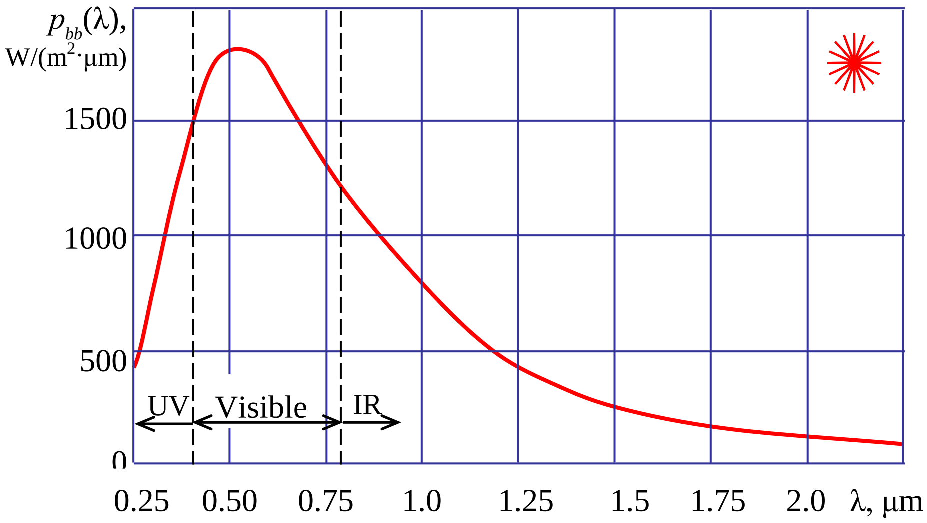

Fig. 7.6. Dependence of the solar irradiance on the wavelength approximated by the blackbody spectrum at T = 5778 K

The spectrum of solar radiation turned out to be very close to the radiation spectrum of a black body at a temperature T = 5778 K (Fig. 7.6). Maximum radiation is observed at a wavelength of 555 nm (green light). Such a spectrum has solar radiation outside the atmosphere, where the radiation flux density is 1365 W/m^2 (the so-called solar constant). Atmospheric absorption creates dips in this spectrum. At summer noon, at the latitude of Kyiv, on the Earth's surface falls the solar radiation flux of 760 W/m^2.

Thermal radiation becomes visible at a temperature of 800 K (a deep red glow of the body appears). As the temperature rises to 1300 K, the radiation turns bright red. At a temperature of 1500 K, the radiation becomes yellow, and at 1800 K changes to dazzling white.

7.4. On the Doorstep of a Quantum Era

The Planck's formula opened the era of quantum physics, the laws of which are very different from the laws of classical physics and often defy logical explanation. In 1920, Planck said in his Nobel lecture [7.1]:

«What becomes of the energy of a photon after complete emission? Does it spread out in all directions with further propagation in the sense of Huygens' wave theory, so constantly taking up more space, in boundless progressive attenuation? Or does it fly out like a projectile in one direction in the sense of Newton's emanation theory? In the first case, the quantum would no longer be in the position to concentrate energy upon a single point in space in such a way as to release an electron from its atomic bond, and in the second case, the main triumph of the Maxwell theory – the continuity between the static and the dynamic fields and, with it, the complete understanding we have enjoyed, until now, of the fully investigated interference phenomena – would have to be sacrificed, both being very unhappy consequences for today's theoreticians».

There is no answer to the question asked by Planck 100 years ago.

7. References

1. Planck, M. The Genesis and Present State of Development of the Quantum Theory. Nobel Lecture [Electron. resource]. – Access link: https://www.nobelprize.org/prizes/physics/1918/planck/lecture/